Probabalistic Parsing

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Statistical Parsing

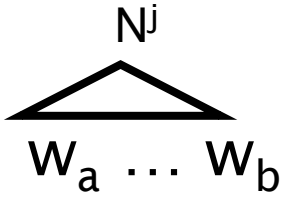
- Statistical parsing uses a probabilistic model of syntax in order to assign probabilities to each parse tree.
- Provides principled approach to resolving syntactic ambiguity.
- Allows supervised learning of parsers from tree-banks of parse trees provided by human linguists.
- Also allows unsupervised learning of parsers from unannotated text, but the accuracy of such parsers has been limited.

Probabilistic Context Free Grammar (PCFG)

- A PCFG is a probabilistic version of a CFG where each production has a probability.
- Probabilities of all productions rewriting a given non-terminal must add to 1, defining a distribution for each non-terminal.
- String generation is now probabilistic where production probabilities are used to non-deterministically select a production for rewriting a given non-terminal.

PCFGs – Notation

- $w_{1n} = w_1 \ldots w_n$ = the word sequence from 1 to n (sentence of length n)
- w_{ab} = the subsequence $w_a \ldots w_b$
- N^j_{ab} = the nonterminal N^j dominating $w_a \ldots w_b$



- We'll write $P(N^i \rightarrow \zeta^j)$ to mean $P(N^i \rightarrow \zeta^j \mid N^i)$
- We'll want to calculate $\max_t P(t \Rightarrow^* w_{ab})$

The probability of trees and strings

- $P(w_{1n}, t)$ -- The probability of tree is the product of the probabilities of the rules used to generate it.

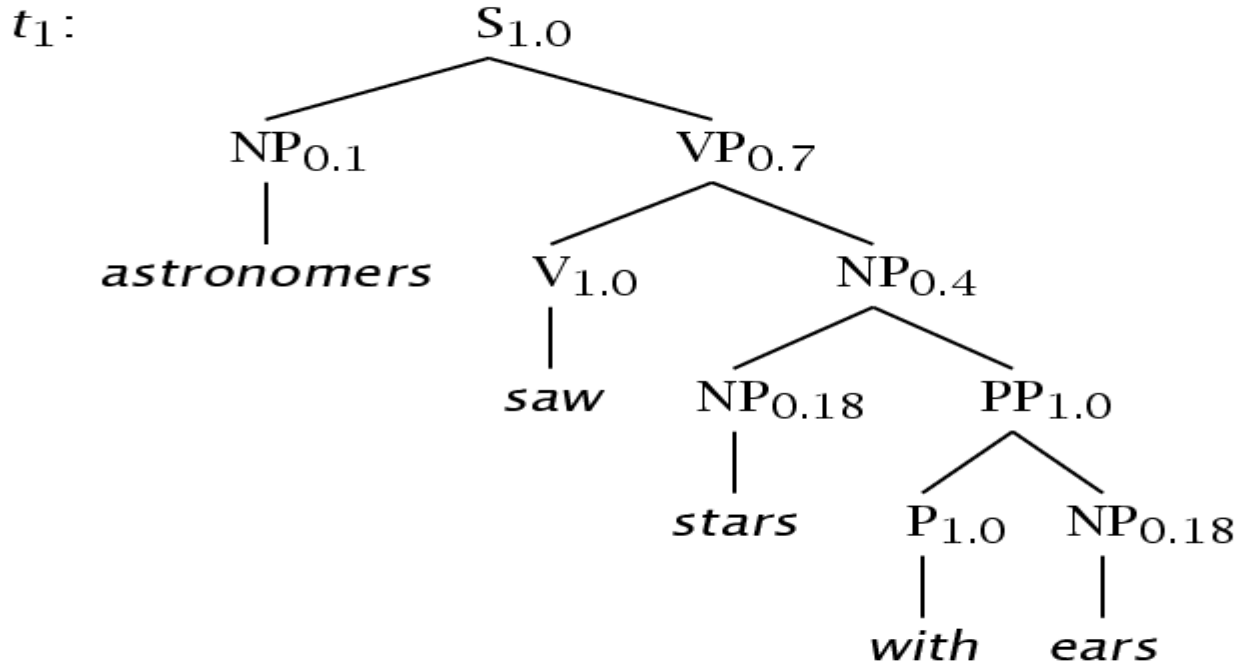
$$P(w_{1n}, t) = \prod_{\{R=X \rightarrow AB\} \in t} P(R) \prod_{\{R=X \rightarrow w_i\} \in t} P(R)$$

- $P(w_{1n})$ -- The probability of the string is the sum of the probabilities of the trees which have that string as their yield
  - $P(w_{1n}) = \Sigma t\ P(w_{1n}, t)$ where t is a parse of w_{1n}

Example: A Simple PCFG
(in Chomsky Normal Form)

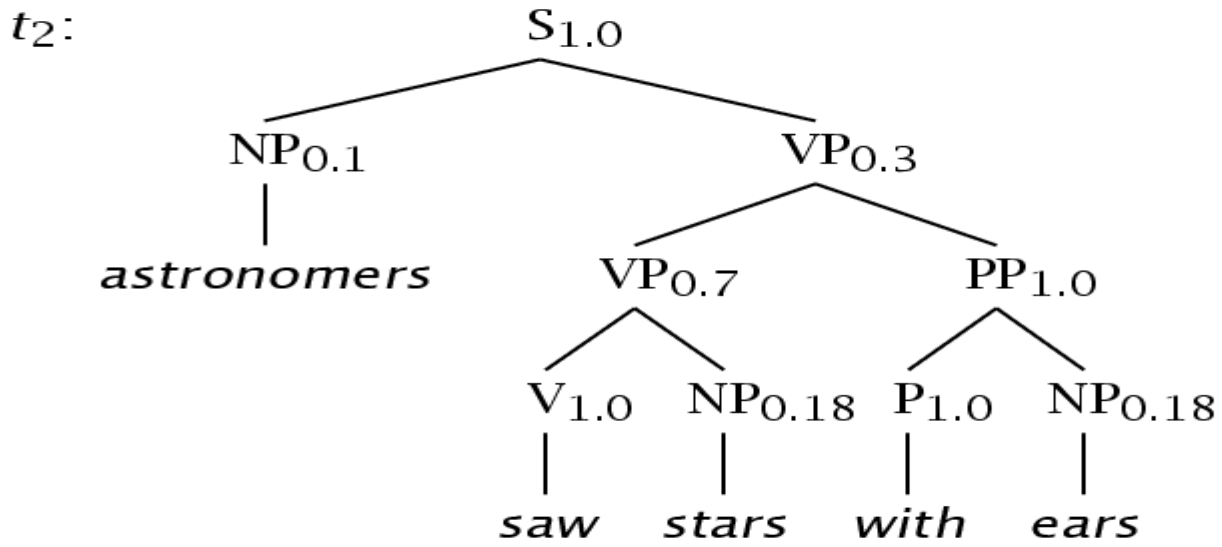
S	→	NP VP	1.0	NP	→	NP PP	0.4
VP	→	V NP	0.7	NP	→	*astronomers*	0.1
VP	→	VP PP	0.3	NP	→	*ears*	0.18
PP	→	P NP	1.0	NP	→	*saw*	0.04
P	→	*with*	1.0	NP	→	*stars*	0.18
V	→	*saw*	1.0	NP	→	*telescope*	0.1

Probability of “stars with ears”



$P(t_1) = 1.0 * 0.1 * 0.7 * 1.0 * 0.4 * 0.18 * 1.0 * 1.0 * 0.18$

Probability of "seeing with ears"



$P(t_2) = 1.0 * 0.1 * 0.3 * .07 * 0.1 * 0.18 * 1.0 * 1.0 * 0.18$

Tree and String Probabilities

w_{15} = astronomers saw stars with ears

P(t1) = 1.0 * 0.1 * 0.7 * 1.0 * 0.4 * 0.18 * 1.0 * 1.0 * 0.18

= 0.0009072

P(t2) = 1.0 * 0.1 * 0.3 * 0.7 * 1.0 * 0.18 * 1.0 * 1.0 * 0.18

= 0.0006804

$P(w_{15})$ = P(t1) + P(t2)

= 0.0009072 + 0.0006804

= 0.0015876

Simple PCFG for ATIS

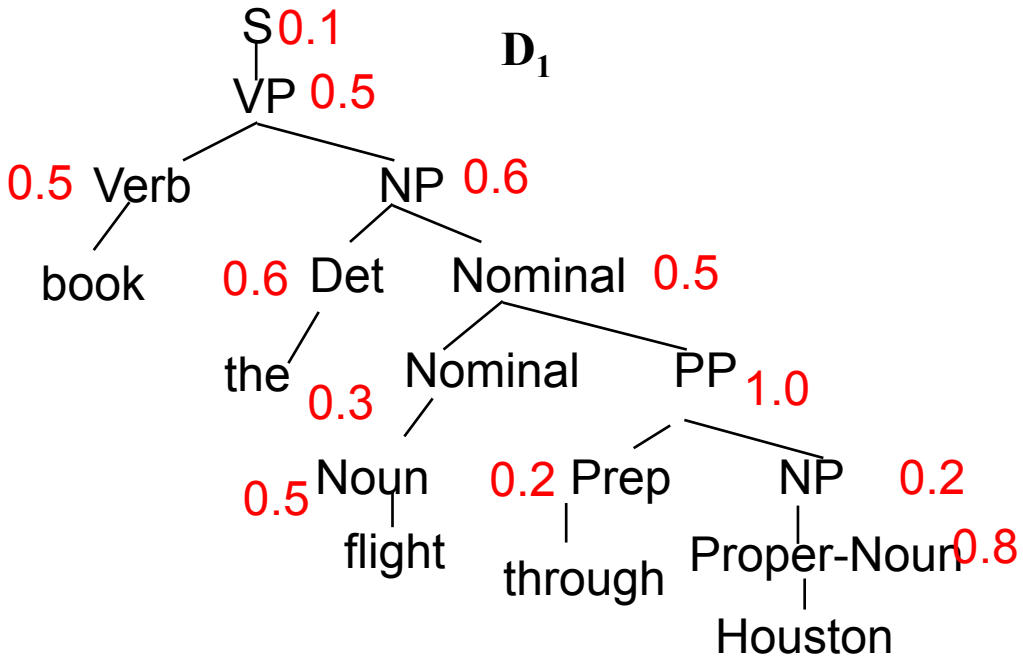
Grammar	Prob	
S → NP VP	0.8	
S → Aux NP VP	0.1	1.0
S → VP	0.1	
NP → Pronoun	0.2	
NP → Proper Noun	0.2	1.0
NP → Det Nominal	0.6	
Nominal → Noun	0.3	
Nominal → Nominal Noun	0.2	1.0
Nominal → Nominal PP	0.5	
VP → Verb	0.2	
VP → Verb NP	0.5	1.0
VP → VP PP	0.3	
PP → Prep NP	1.0	

Lexicon
Det → the | a | that | this
0.6 0.2 0.1 0.1
Noun → book | flight | meal | money
0.1 0.5 0.2 0.2
Verb → book | include | prefer
0.5 0.2 0.3
Pronoun → I | he | she | me
0.5 0.1 0.1 0.3
Proper-Noun → Houston | NWA
0.8 0.2
Aux → does
1.0
Prep → from | to | on | near | through
0.25 0.25 0.1 0.2 0.2

Sentence Probability

- Assume productions for each node are chosen independently.
- Probability of derivation is the product of the probabilities of its productions.

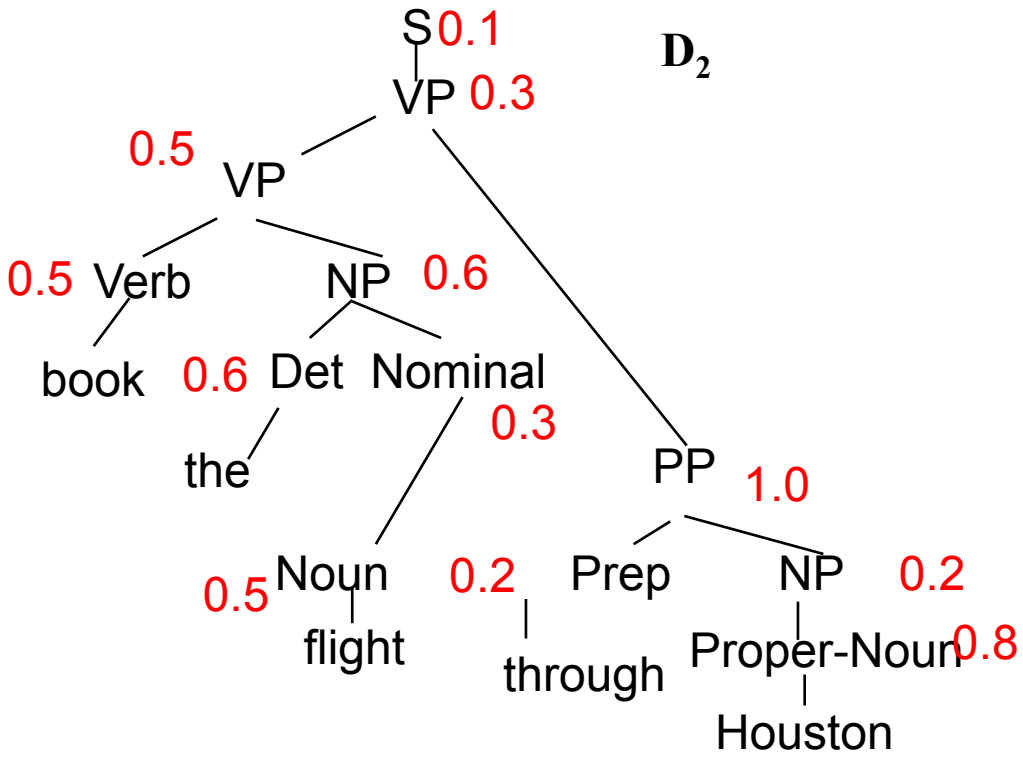
$P(D_1)$ = 0.1 x 0.5 x
0.5 x 0.6 x
0.6 x 0.5 x
0.3 x 1.0 x
0.5 x 0.2 x 0.2 x
0.8
= 0.0000216



Sentence Probability

- Resolve ambiguity by picking most probable parse tree.

$P(D_1)$ = 0.1 x 0.3 x
0.5 x
0.5 x 0.6 x
0.6 x 0.3 x
1.0 x
0.5 x 0.2 x 0.2 x
0.8
= 0.00001296



D_2

Sentence Probability

- Probability of a sentence is the sum of the probabilities of all of its derivations.

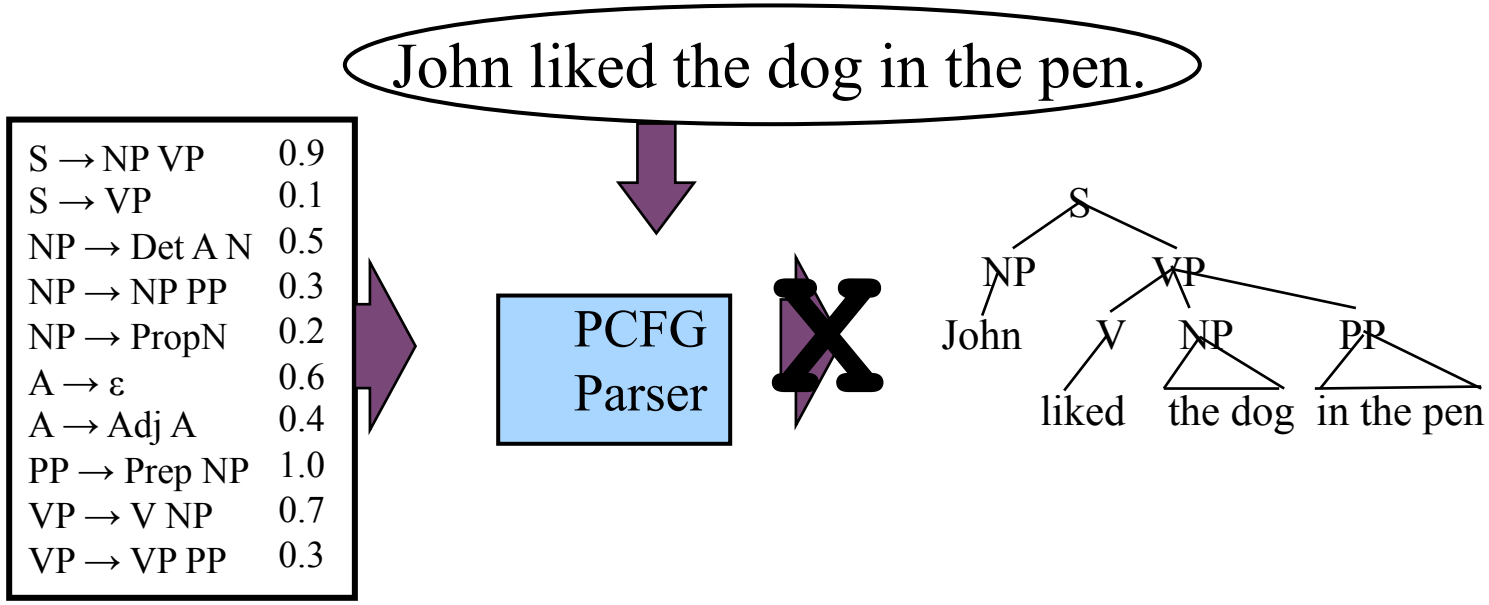
P("book the flight through Houston") =
$P(D_1) + P(D_2) = 0.0000216 + 0.00001296$
$= 0.00003456$

Three Useful PCFG Tasks

- Observation likelihood: To classify and order sentences.
- Most likely derivation: To determine the most likely parse tree for a sentence.
- Maximum likelihood training: To train a PCFG to fit empirical training data.

PCFG: Most Likely Derivation

- There is an analog to the Viterbi algorithm to efficiently determine the most probable derivation (parse tree) for a sentence.



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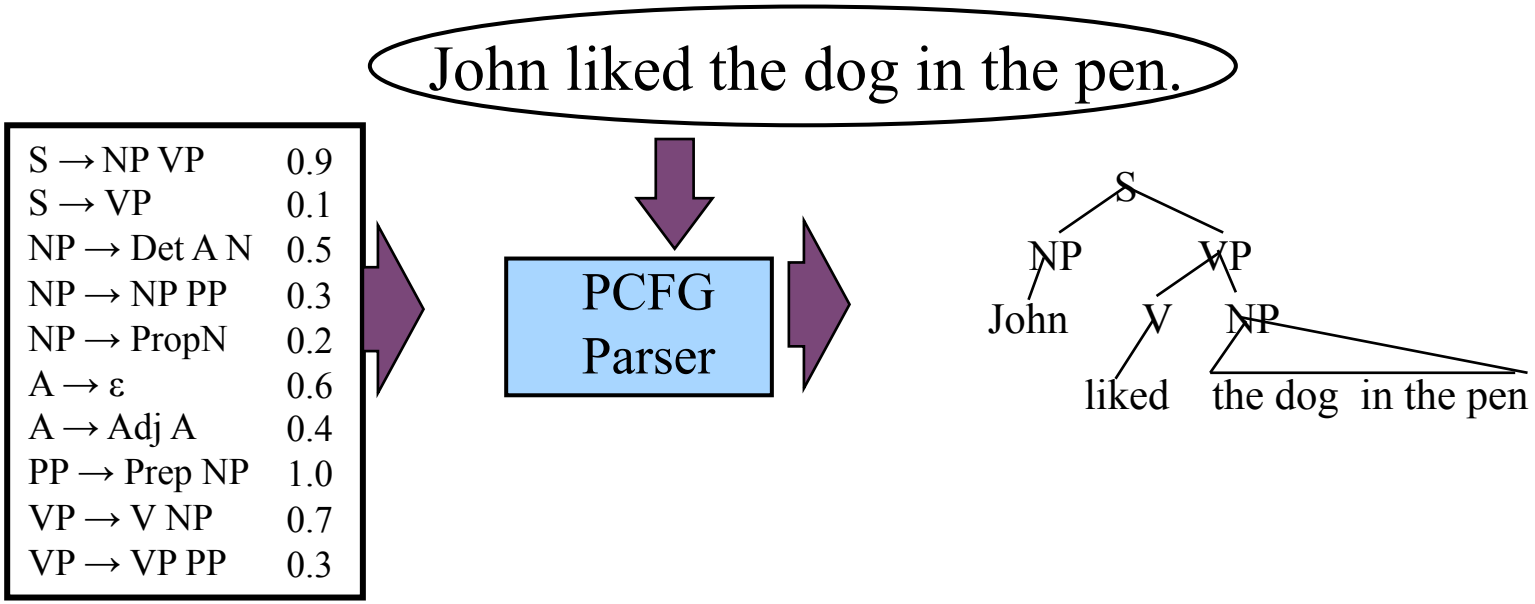

Probabilistic CKY

- CKY can be modified for PCFG parsing by including in each cell a probability for each non-terminal.
- Cell[i,j] must retain the most probable derivation of each constituent (non-terminal) covering words i +1 through j together with its associated probability.
- When transforming the grammar to CNF, must set production probabilities to preserve the probability of derivations.

CKY Parsing

A worked example

The grammar: Binary, no epsilons,

S → NP VP	0.9
S → VP	0.1
VP → V NP	0.5
VP → V	0.1
VP → V @VP_V	0.3
VP → V PP	0.1
@VP_V → NP PP	1.0
NP → NP NP	0.1
NP → NP PP	0.2
NP → N	0.7
PP → P NP	1.0

N → *people*	0.5
N → *fish*	0.2
N → *tanks*	0.2
N → *rods*	0.1
V → *people*	0.1
V → *fish*	0.6
V → *tanks*	0.3
P → *with*	1.0

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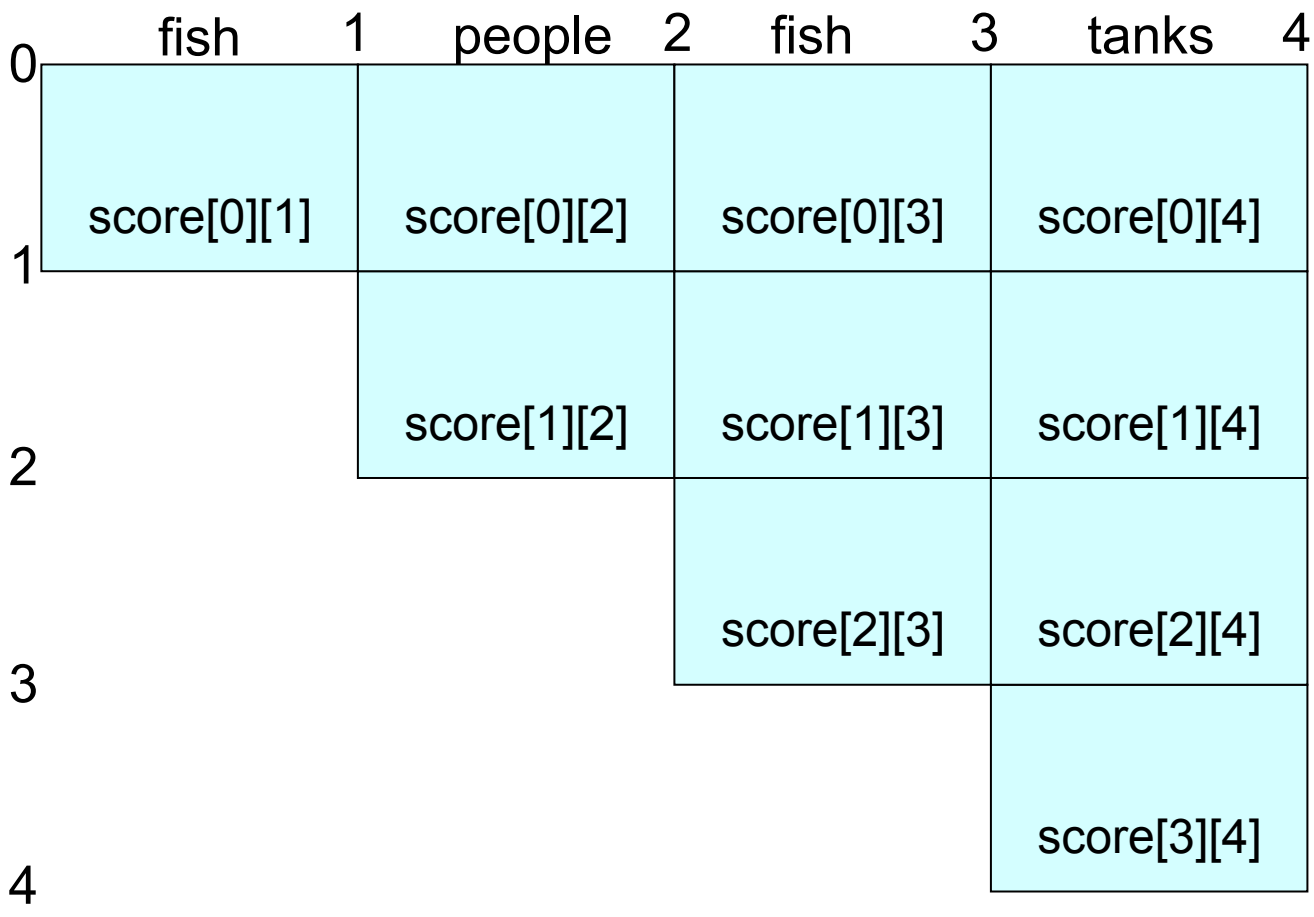
P → *with*	1.0



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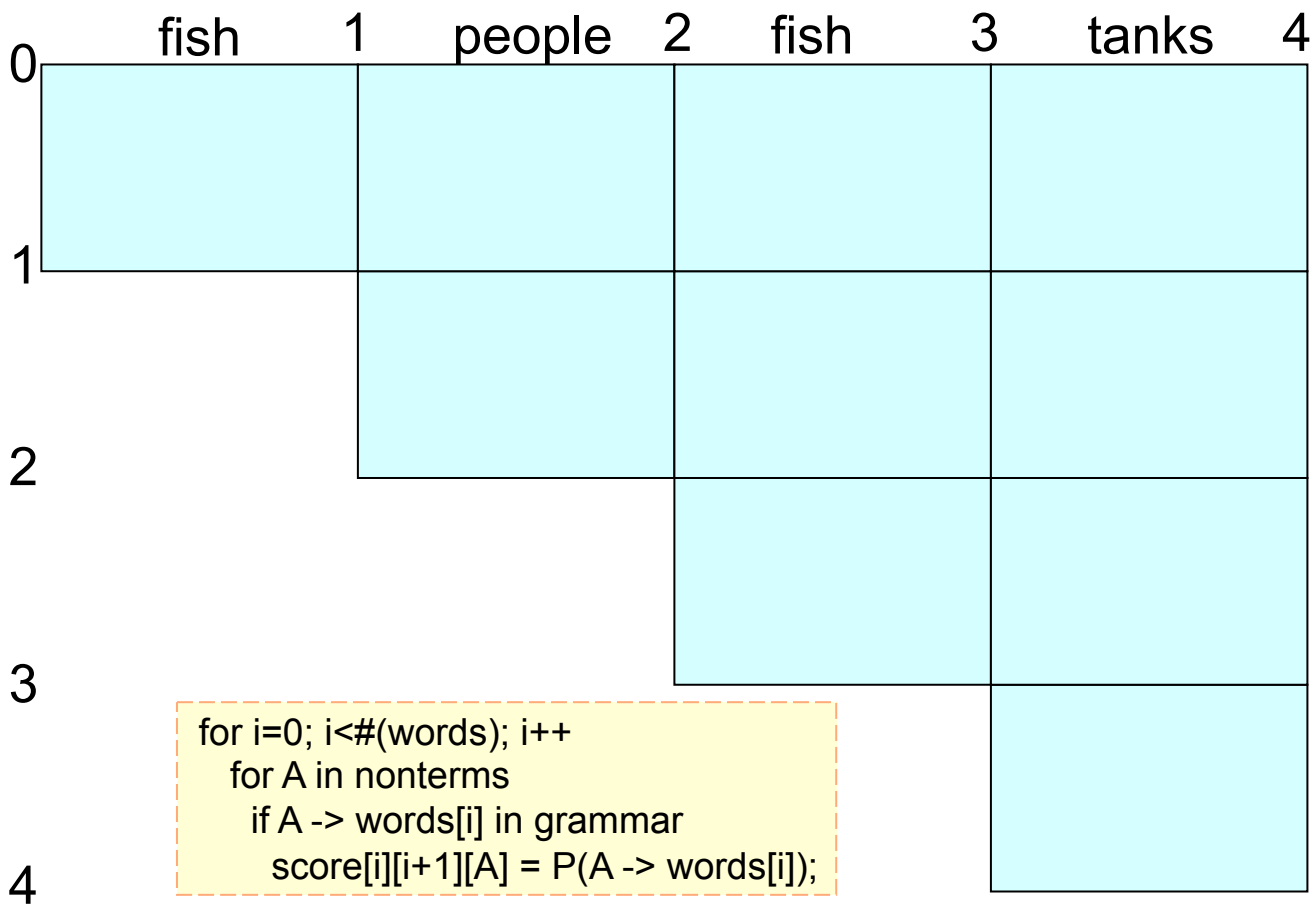


```
for i=0; i<#(words); i++
   for A in nonterms
     if A -> words[i] in grammar
       score[i][i+1][A] = P(A -> words[i]);
```

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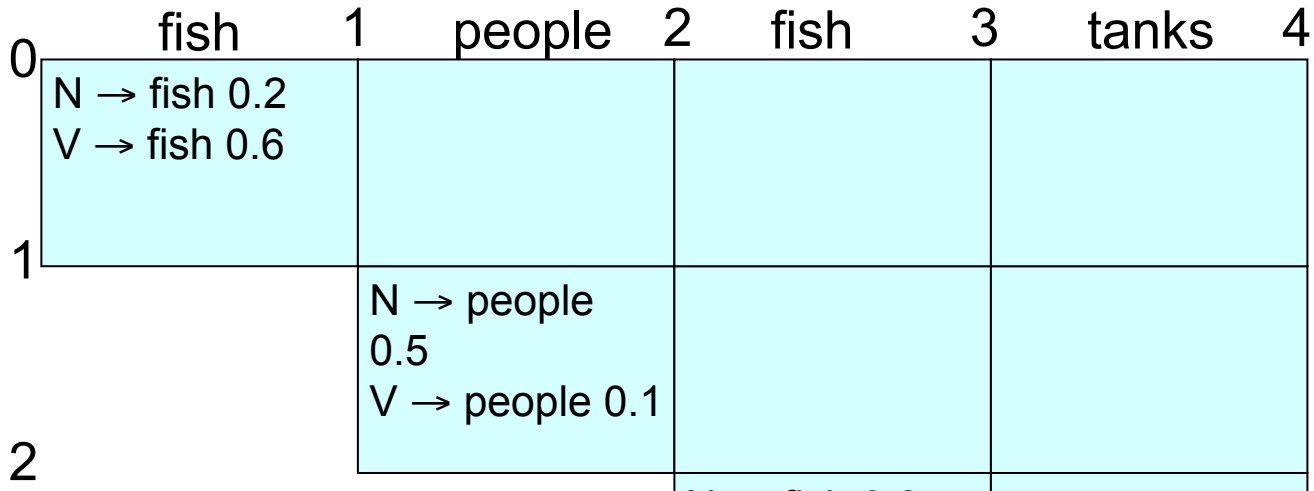


```
// handle unaries
boolean added = true
    while added
      added = false
      for A, B in nonterms
        if score[i][i+1][B] > 0 && A->B in grammar
          prob = P(A->B)*score[i][i+1][B]
          if(prob > score[i][i+1][A])
            score[i][i+1][A] = prob
            back[i][i+1][A] = B
            added = true
```

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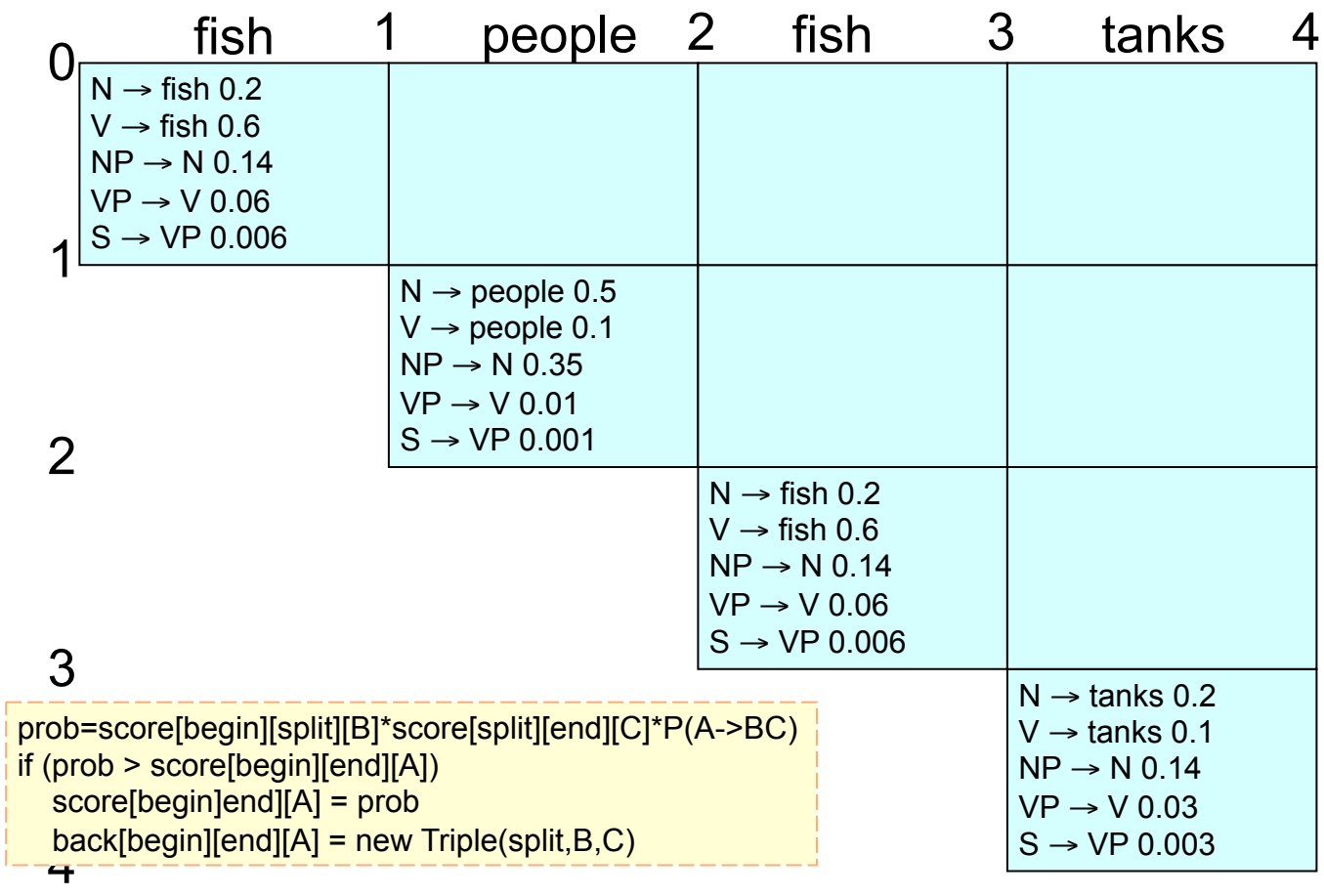

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	0 fish 1	1 people 2	2 fish 3	3 tanks 4
0	N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006	NP → NP NP 0.0049 VP → V NP 0.105 S → NP VP 0.00126		
1		N → people 0.5 V → people 0.1 NP → N 0.35 VP → V 0.01 S → VP 0.001	NP → NP NP 0.0049 VP → V NP 0.007 S → NP VP 0.0189	
2			N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006	NP → NP NP 0.00196 VP → V NP 0.042 S → NP VP 0.00378
3				N → tanks 0.2 V → tanks 0.1 NP → N 0.14 VP → V 0.03 S → VP 0.003
4				

```
//handle unaries
boolean added = true
while added
  added = false
  for A, B in nonterms
    prob = P(A->B)*score[begin][end][B];
    if prob > score[begin][end][A]
      score[begin][end][A] = prob
      back[begin][end][A] = B
      added = true
```

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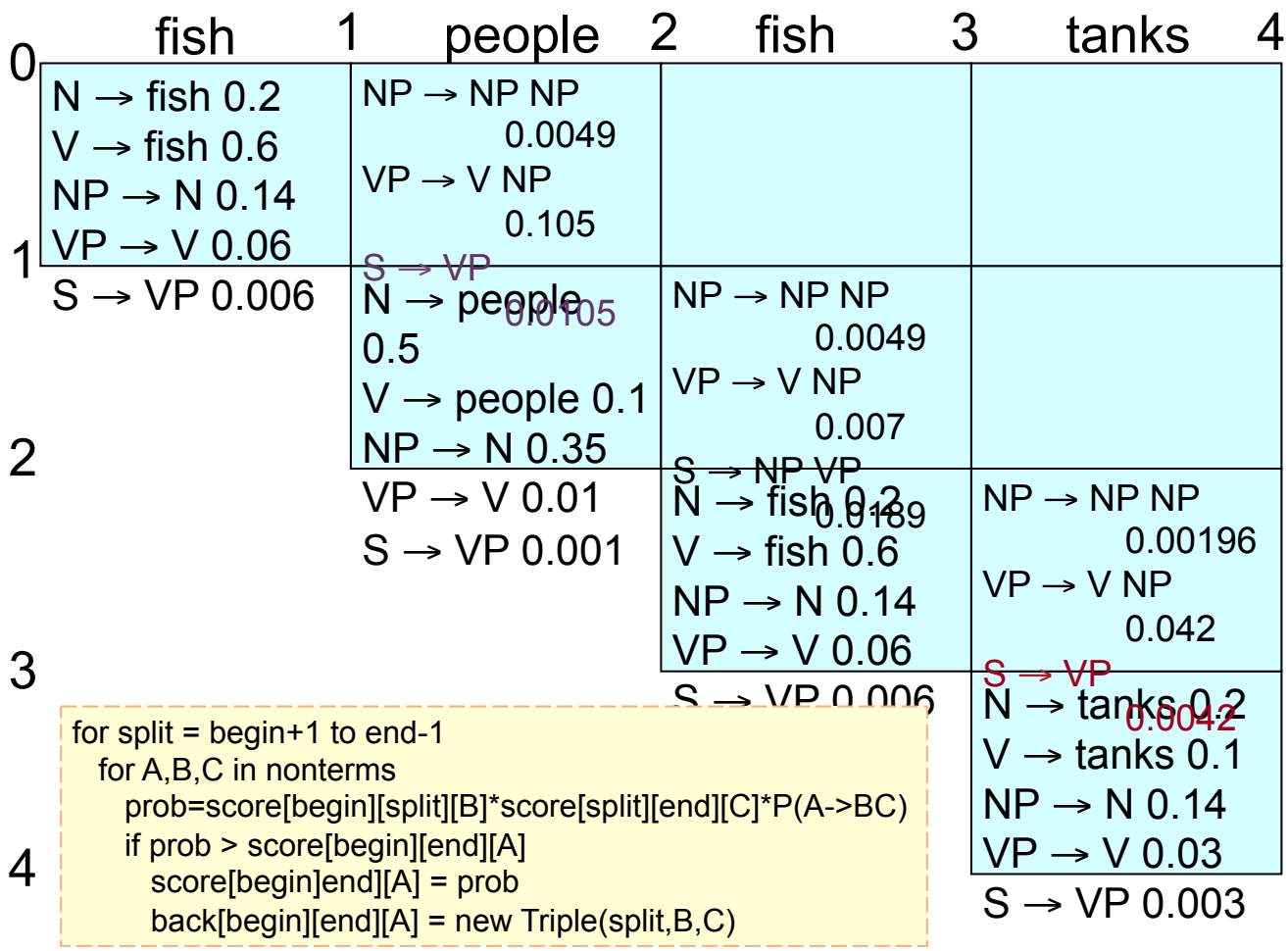

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V → *fish*	0.6
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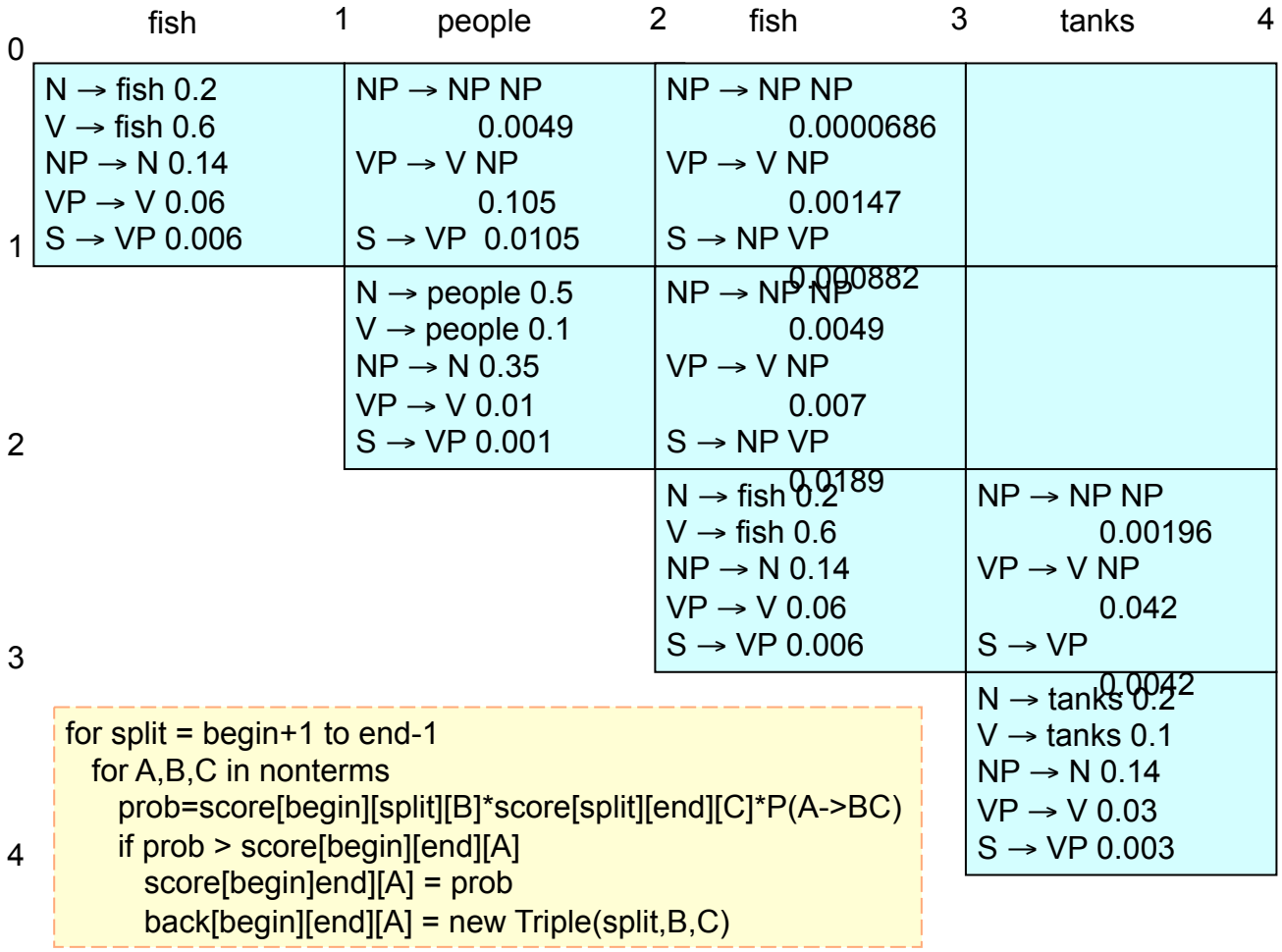

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VP → V PP	0.1
@VP_V → NP PP	1.0
NP → NP NP	0.1
NP → NP PP	0.2
NP → N	0.7
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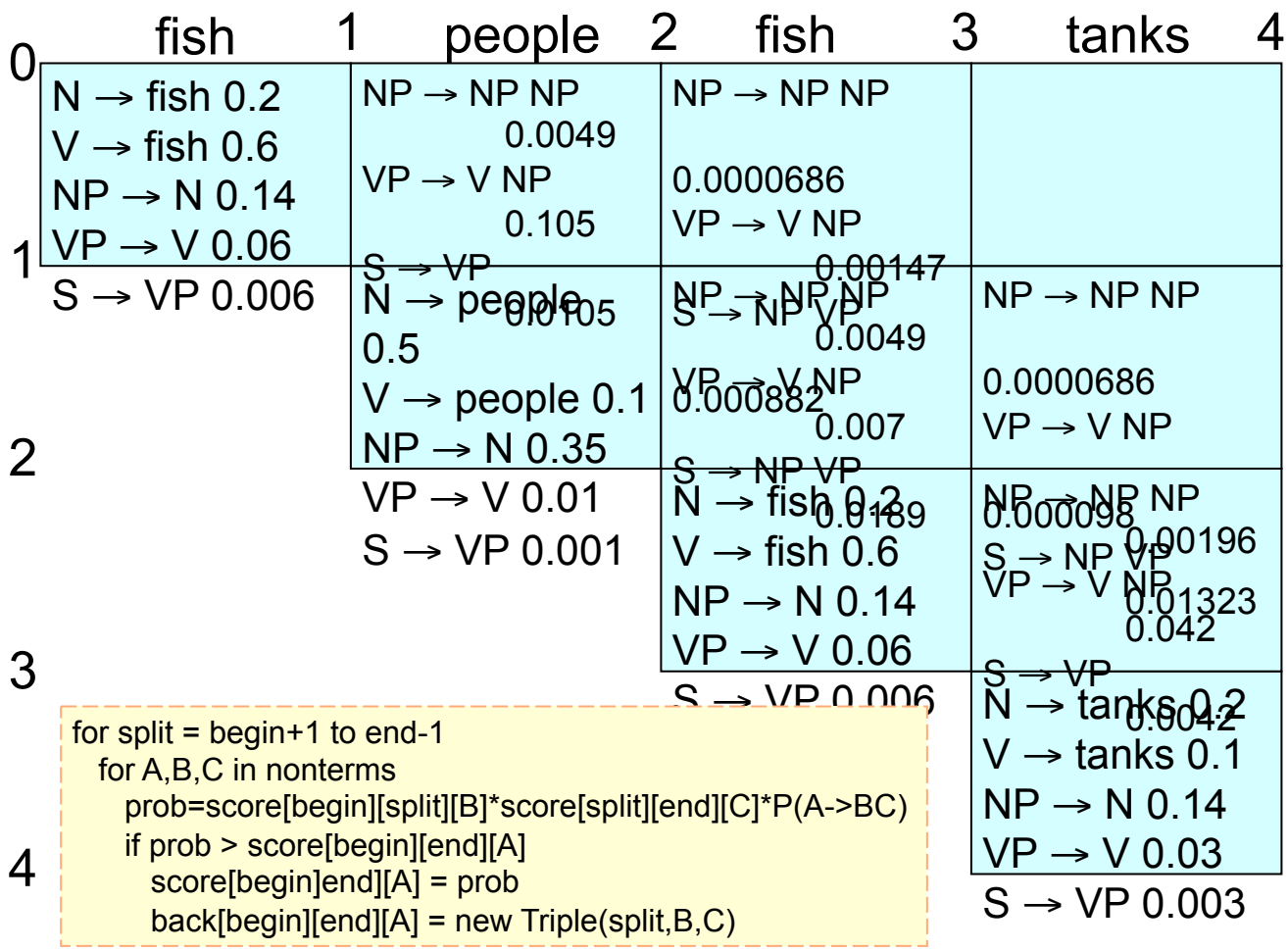
N → *tanks*	0.2
N → *rods*	0.1
V → *people*	0.1
V → *fish*	0.6
V → *tanks*	0.3
P → *with*	1.0



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N → *tanks*	0.2
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1			N → people 0.5 V → people 0.1 NP → N 0.35 VP → V 0.01 S → VP 0.001		NP → NP NP 0.0049 VP → V NP 0.007 S → NP VP 0.0189		NP → NP NP 0.000068 VP → V NP 0.000098 S → NP VP 0.01323	
2					N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006		NP → NP NP 0.00196 VP → V NP 0.042 S → VP 0.0042	
3							N → tanks 0.2 V → tanks 0.1 NP → N 0.14 VP → V 0.03 S → VP 0.003	
4								

Call buildTree(score, back) to get the best parse