Basic Text Processing

Finite State Automata

+ Models and Algorithms

- By models we mean the formalisms that are used to capture the various kinds of linguistic knowledge we need.
- Algorithms are then used to manipulate the knowledge representations needed to tackle the task at hand.

Models

- State machines
- Rule-based approaches
- Logical formalisms
- Probabilistic models

+ Algorithms

- Many of the algorithms that we'll study will turn out to be transducers; algorithms that take one kind of structure as input and output another.
- Unfortunately, ambiguity makes this process difficult. This leads us to employ algorithms that are designed to handle ambiguity of various kinds

+ Paradigms

- State-space search
  - To manage the problem of making choices during processing when we lack the information needed to make the right choice
- Dynamic programming
  - To avoid having to redo work during the course of a state-space search
    - CKY, Earley, Minimum Edit Distance, Viterbi, Baum-Welch
- Classifiers
  - Machine learning based classifiers that are trained to make decisions based on features extracted from the local context

Theory of Computation: A Historical Perspective

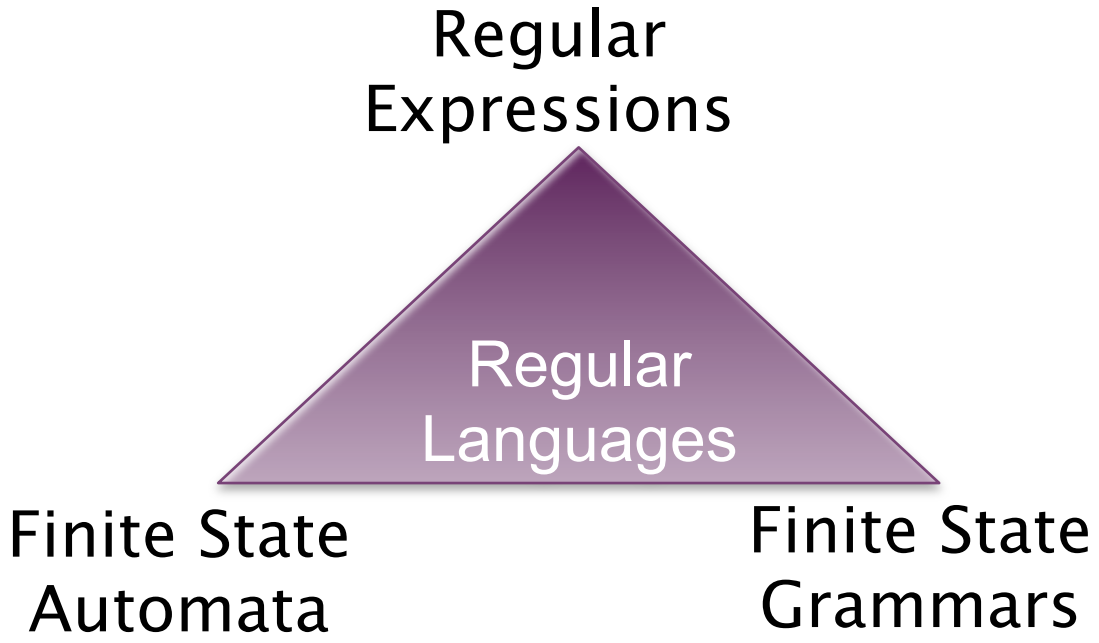
1930s	• Alan Turing studies Turing machines • Decidability • Halting problem
1940-1950s	• "Finite automata" machines studied • Noam Chomsky proposes the "Chomsky Hierarchy" for formal languages
1969	Cook introduces "intractable" problems or "NP-Hard" problems
1970-	Modern computer science: compilers, computational & complexity theory evolve

Summary on REs so far

- Everybody does it
  - Emacs, vi, perl, grep, etc..
- Regular expressions are a compact textual representation of a set of strings representing a language.
- Regular expressions are perhaps the single most useful tool for text manipulation
- Dumb but ubiquitous Eliza: you can do a lot with simple regular-expression substitutions

Three Views

- Three equivalent formal ways to look at what we're up to



Finite State Automata

- Regular expressions can be viewed as a textual way of specifying the structure of finite-state automata.
- FSAs and their probabilistic relatives are at the core of much of what we'll be doing all semester.
- They also capture significant aspects of what linguists say we need for morphology and parts of syntax.

Languages & Grammars

Or "words"

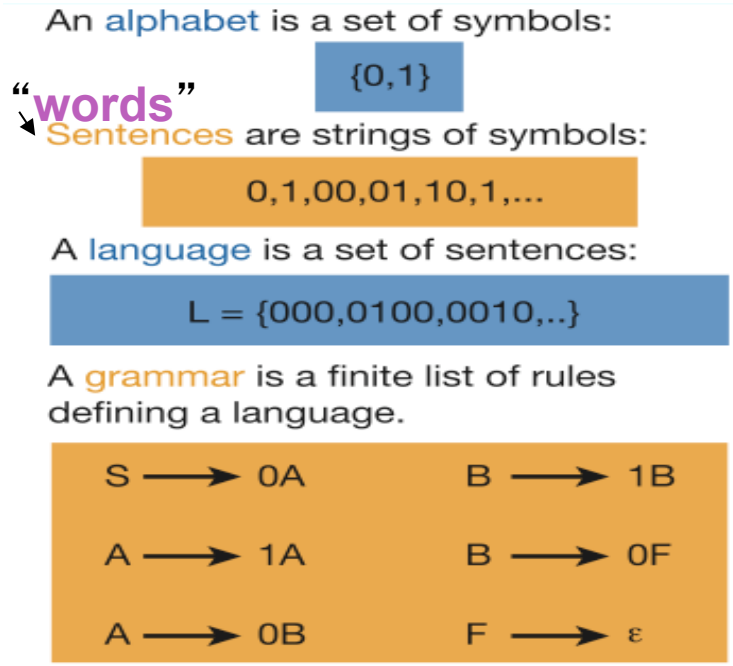


Image source: Nowak et al. Nature, vol 417, 2002

- Languages: "*A language is a collection of sentences of finite length all constructed from a finite alphabet of symbols*"
- Grammars: "*A grammar can be regarded as a device that enumerates the sentences of a language*" - nothing more, nothing less
- *N. Chomsky, Information and Control, Vol 2, 1959*

Alphabet

An alphabet is a finite, non-empty set of symbols

- We use the symbol ∑ (sigma) to denote an alphabet
- Examples:
  - Binary: ∑ = {0,1}
  - All lower case letters: ∑ = {a,b,c,..z}
  - Alphanumeric: ∑ = {a-z, A-Z, 0-9}
  - DNA molecule letters: ∑ = {a,c,g,t}
  - …

Strings

A string or word is a finite sequence of symbols chosen from ∑

- ***Empty string is ε (or "epsilon")***

- Length of a string *w,* denoted by "|*w*|", is equal to the *number of (non- ε) characters in the string*
  - *E.g., x = 010100* |*x*| = *6*
  - *x = 01 ε 0 ε 1 ε 00 ε* |*x*| = *?*

  - *xy* = concatentation of two strings *x* and *y*

Powers of an alphabet

Let $\sum$ be an alphabet.

- $\sum^k$ = the set of all strings of length k
- $\sum^* = \sum^0 \cup \sum^1 \cup \sum^2 \cup \ldots$
- $\sum^+ = \sum^1 \cup \sum^2 \cup \sum^3 \cup \ldots$

Languages

*L is a said to be a language over alphabet ∑, only if L ⊆ ∑**

➔ this is because ∑* is the set of all strings (of all possible length including 0) over the given alphabet ∑

Examples:

1. Let L be *the* language of all strings consisting of *n* 0's followed by *n* 1's:
   L = {ε,01,0011,000111,…}
2. Let L be *the* language of all strings of with equal number of 0's and 1's:
   L = {ε,01,10,0011,1100,0101,1010,1001,…}

Definition: Ø denotes the Empty language

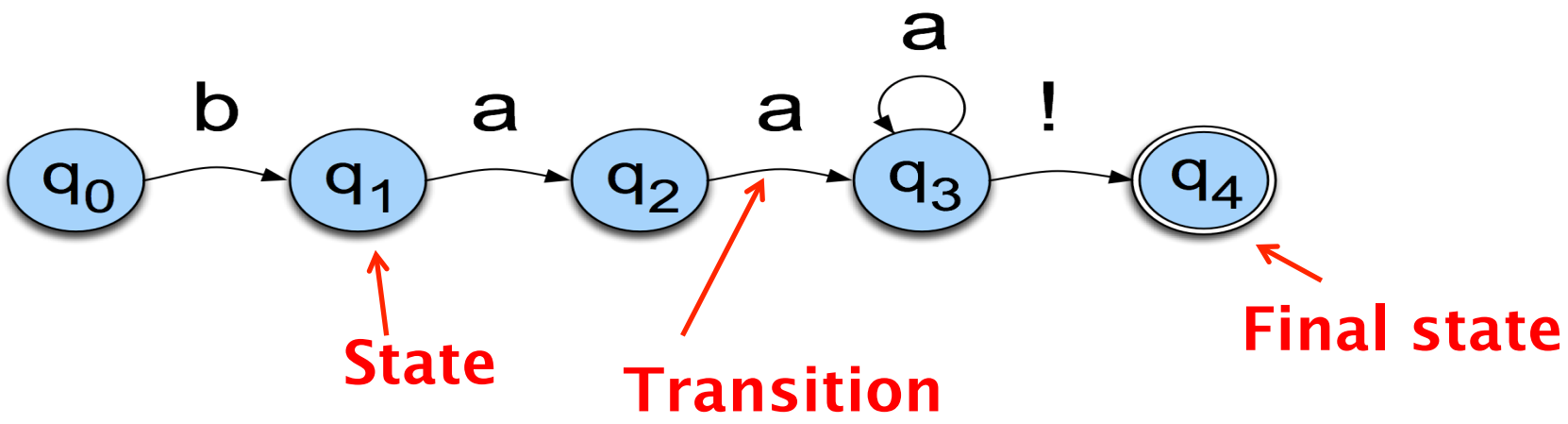
- Let L = {ε}; Is L=Ø?

NO

FSAs as Graphs

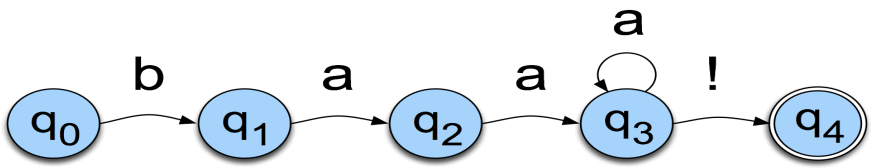
- Let's start with the sheep language

  `/baa+!/`



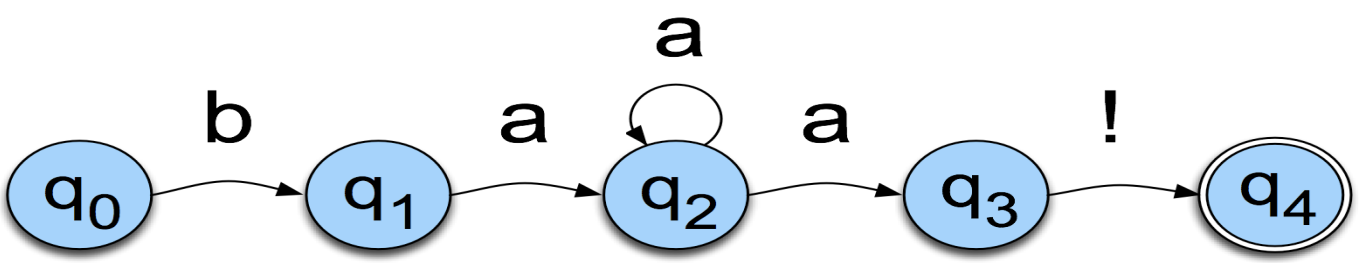
Sheep FSA

- We can say the following things about this machine
  - It has 5 states
  - b, a, and ! are in its alphabet
  - q_0 is the start state
  - q_4 is an accept state
  - It has 5 transitions



But Note

- There are other machines that correspond to this same language



- More on this one later

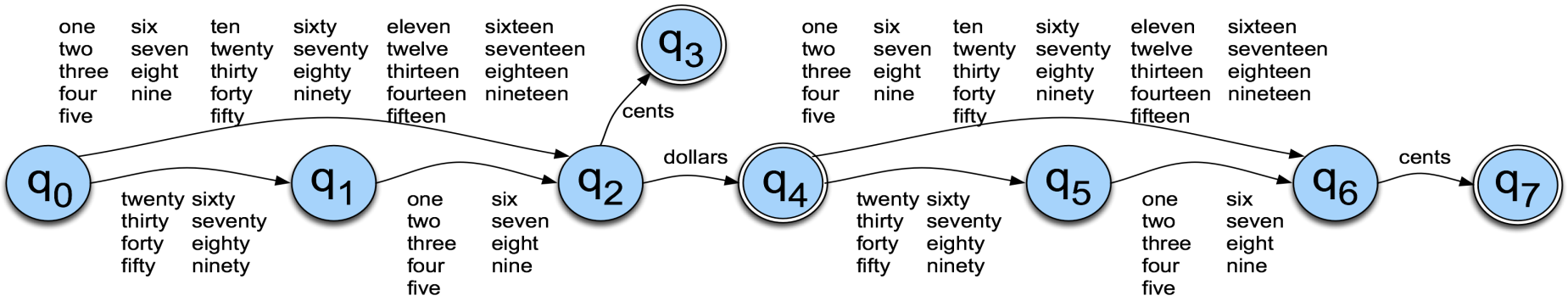
More Formally

- You can specify an FSA by enumerating the following things.
  - The set of states: Q
  - A finite alphabet: Σ
  - A start state
  - A set of accept/final states
  - A transition function that maps QxΣ to Q

About Alphabets

- Don't take term *alphabet* word too narrowly; it just means we need a finite set of symbols in the input.
- These symbols can and will stand for bigger objects that can have internal structure.

Dollars and Cents

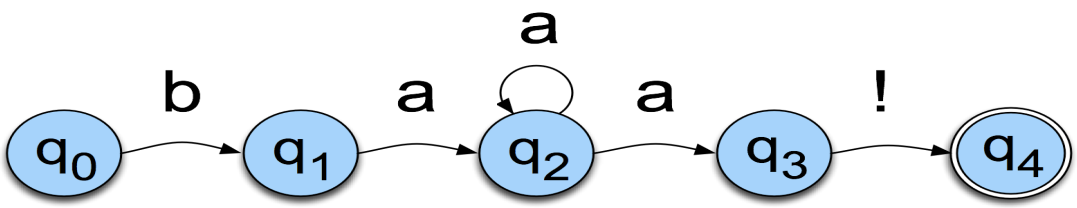


Yet Another View

- The guts of FSAs can ultimately be represented as tables

If you're in state 1 and you see an a, go to state 2

	b	a	!	e
0	1			
1		2		
2		2,3		
3			4	
4				

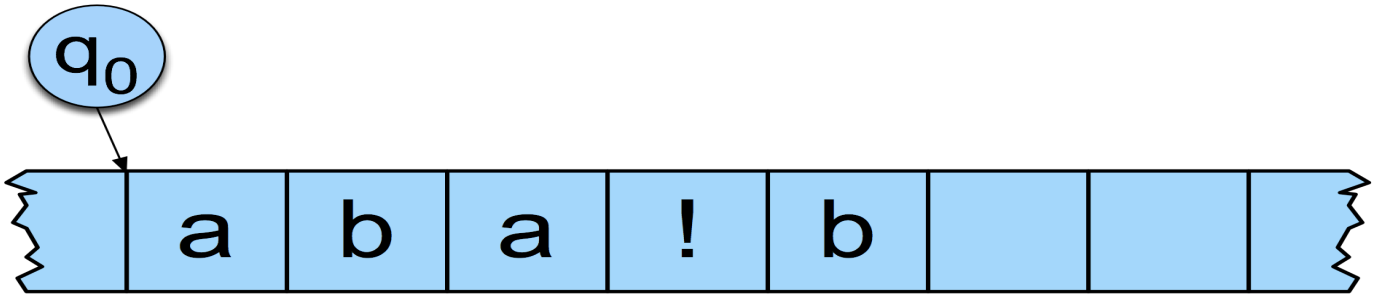


Recognition

- Recognition is the process of determining if a string should be accepted by a machine
- Or… it's the process of determining if a string is in the language we're defining with the machine
- Or… it's the process of determining if a regular expression matches a string
- Those all amount the same thing in the end

Recognition

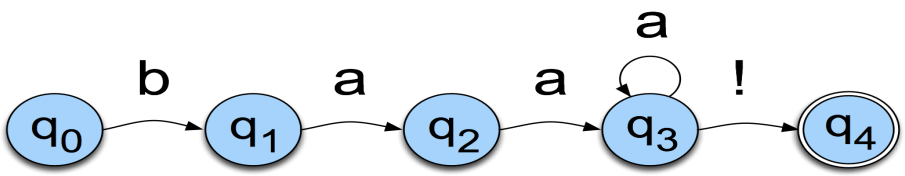
- Traditionally, (Turing's notion) this process is depicted with a tape.



Recognition

- Simply a process of starting in the start state
- Examining the current input
- Consulting the table
- Going to a new state and updating the tape pointer.
- Until you run out of tape.

+ Input tape



	b	a	!	e
0	1			
1		2		
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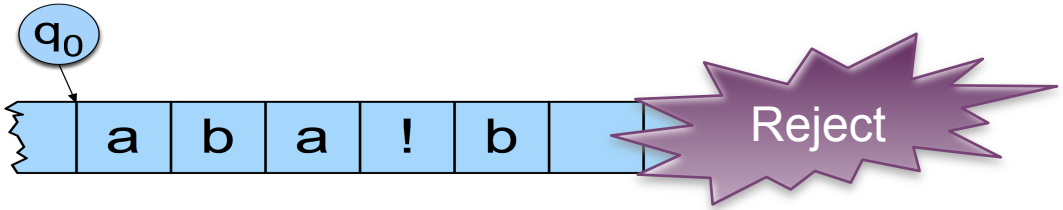
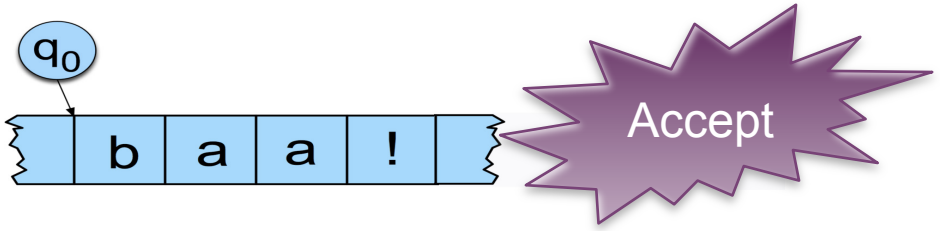
3			4	
4				





+ D-Recognize

```
function D-RECOGNIZE(tape, machine) returns accept or reject

  index ← Beginning of tape
  current-state ← Initial state of machine
  loop
   if End of input has been reached then
     if current-state is an accept state then
        return accept
     else
        return reject
   elsif transition-table[current-state,tape[index]] is empty then
        return reject
   else
        current-state ← transition-table[current-state,tape[index]]
        index ← index + 1
  end
```

Key Points

- Deterministic means that at each point in processing there is always one unique thing to do (no choices).
- D-recognize is a simple table-driven interpreter
- The algorithm is universal for all unambiguous regular languages.
  - To change the machine, you simply change the table.

+ Key Points

- Crudely therefore… matching strings with regular expressions (a la Perl, grep, etc.) is a matter of
  - translating the regular expression into a machine (a table) and
  - passing the table and the string to an interpreter

Recognition as Search

- You can view this algorithm as a trivial kind of state-space search.
- States are pairings of tape positions and state numbers.
- Operators are compiled into the table
- Goal state is a pairing with the end of tape position and a final accept state
- It is trivial because?

No ambiguity

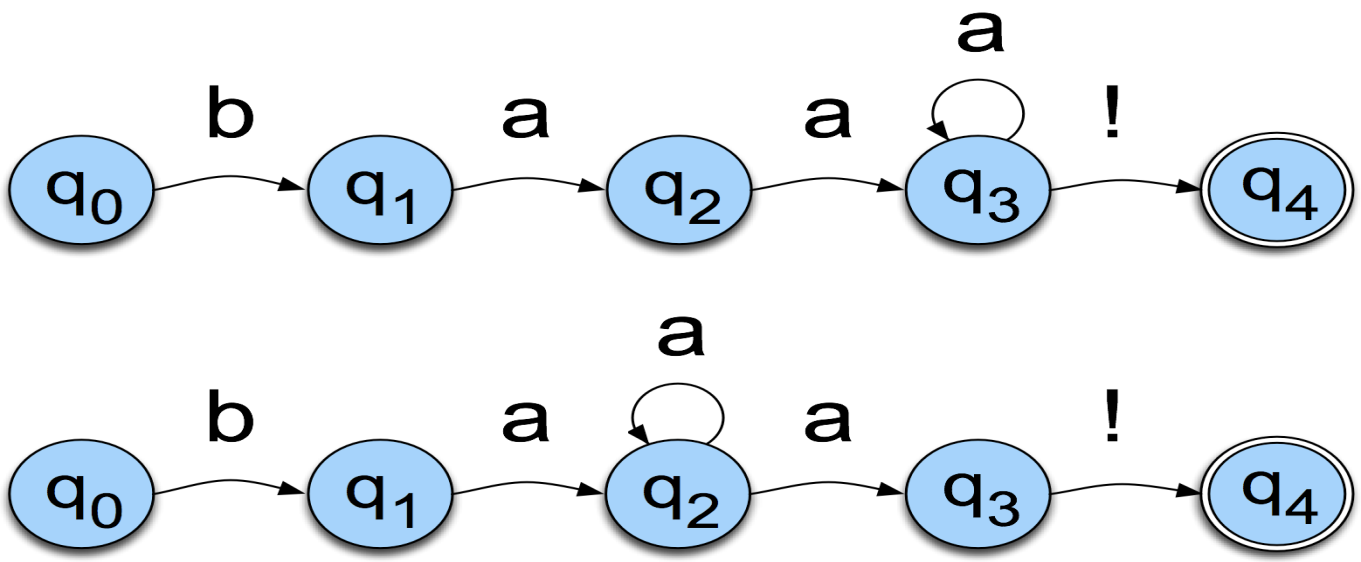
+ Generative Formalisms

- *Formal Languages* are sets of strings composed of symbols from a finite set of symbols.
- Finite-state automata define formal languages (without having to enumerate all the strings in the language)
- The term *Generative* is based on the view that you can run the machine as a generator to get strings from the language.

Generative Formalisms

- FSAs can be viewed from two perspectives:
  - Acceptors that can tell you if a string is in the language
  - Generators to produce *all and only* the strings in the language

+ Non-Determinism



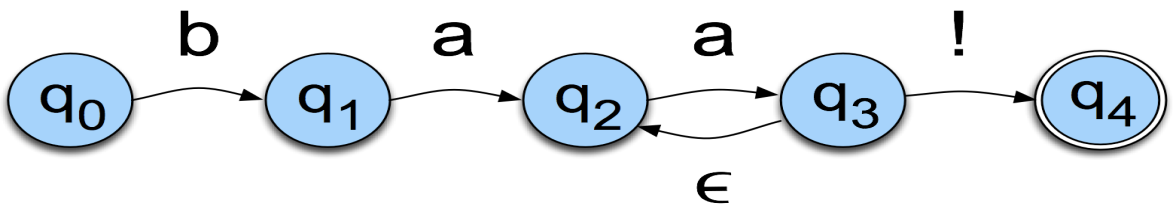
Both define /baa+/

Input: b a a

Stay in q2 or go to q3?

Non-Determinism cont.

- Yet another technique
  - Epsilon transitions
  - Key point: these transitions do not examine or advance the tape during recognition



Equivalence

- Non-deterministic machines can be converted to deterministic ones with a fairly simple construction
  - That means that they have the same power:
  - non-deterministic machines are not more powerful than deterministic ones in terms of the languages they can accept
- Two basic approaches to ND recognition (used in all major implementations of regular expressions)
  1. Either take a ND machine and convert it to a D machine and then do recognition with that.
  2. Or explicitly manage the process of recognition as a state-space search (leaving the machine as is).

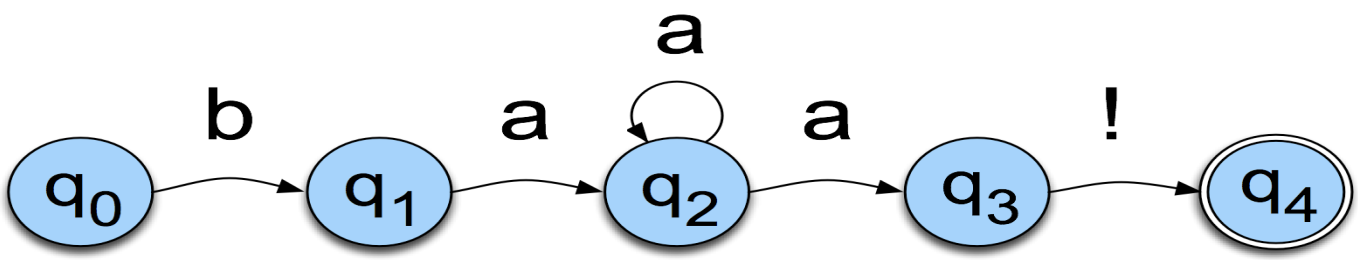
+ Non-Deterministic Recognition: Search

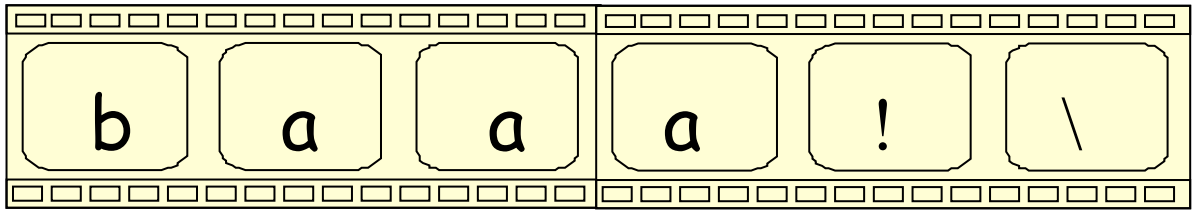
- In a ND FSA *there exists at least one path* through the machine for a string that is in the language defined by the machine.
- *But not all paths* directed through the machine for an accept string lead to an accept state.
- *No paths* through the machine lead to an accept state for a string not in the language.

Non-Deterministic Recognition

- So *success* in non-deterministic recognition occurs when a path is found through the machine that ends in an accept.
- *Failure* occurs when all of the possible paths for a given string lead to failure.

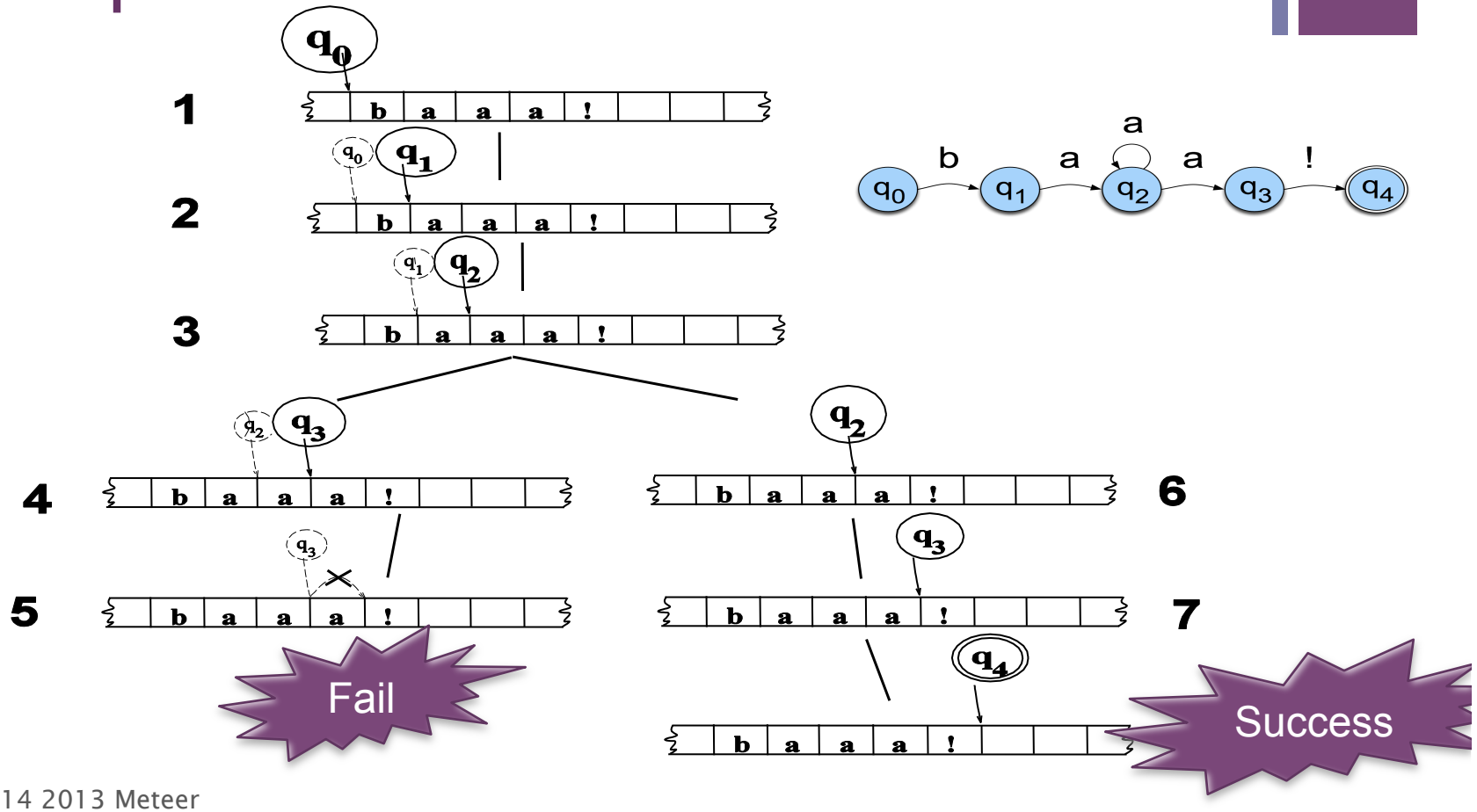
+ Example





q_0 q_1 q_2 q_2 q_3 q_4

+ Example



+ Key Points

- States in the search space are pairings of tape positions and states in the machine.
- By keeping track of as yet unexplored states, a recognizer can systematically explore all the paths through the machine given an input.

Why Bother?

- Non-determinism doesn't get us more formal power and it causes headaches so why bother?
  - More natural (understandable) solutions

Non-determinism

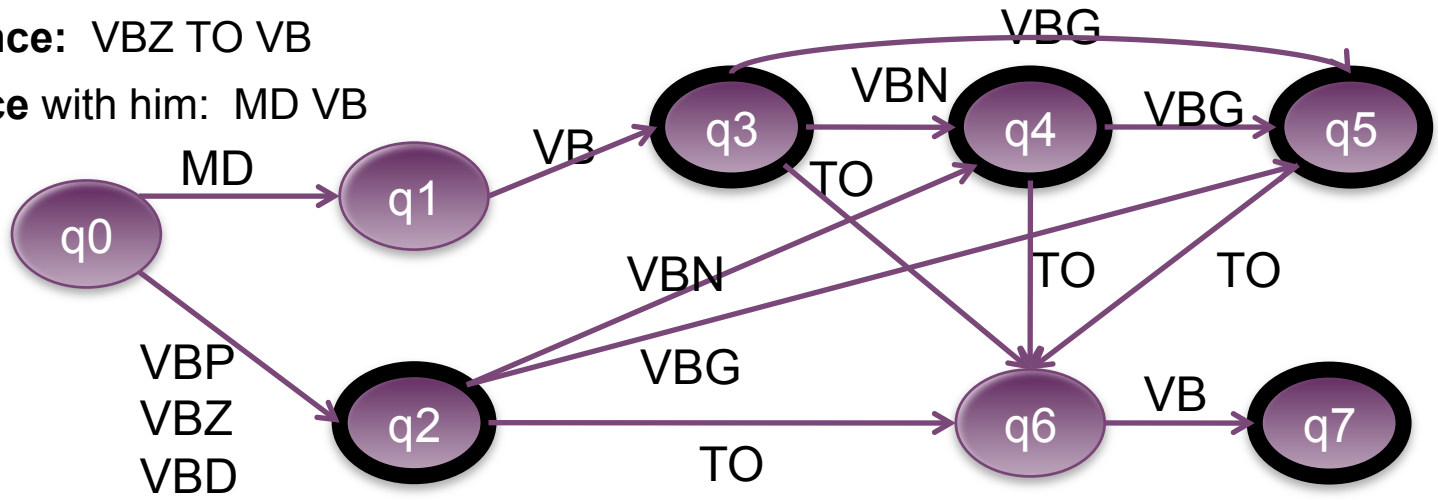
- Three ways to handle this:
  - Backup
  - Look ahead
  - Parallelism
- “Recognition” is search
  - Breadth first
  - Depth First
- Deterministic & nondeterministic equivalent
  - NFSA generally much cleaner
  - DFSA can have many more states
  - See textbook for discussion

Another FSA Example: Verb Groups in English

Tag	Part of speech	Example
MD	Modal	Could, would, will, might, …
VB	Verb base	Eat
VBD	Verb, past tense	Ate (with any subject) (I ate, he ate …)
VBZ	Verb, 3sng, pres	He eats (only he, she, it)
VBP	Verb, non-3sng, pres	I eat, We eat, they eat, you eat
VBG	Verb, gerund	I am eating (always with a form of “is”)
VBN	Verb, past participle	I have eaten (always with a form of “have”)
TO	“to”	“to” when marking a verb as in “to eat”

FSA for Verb Groups

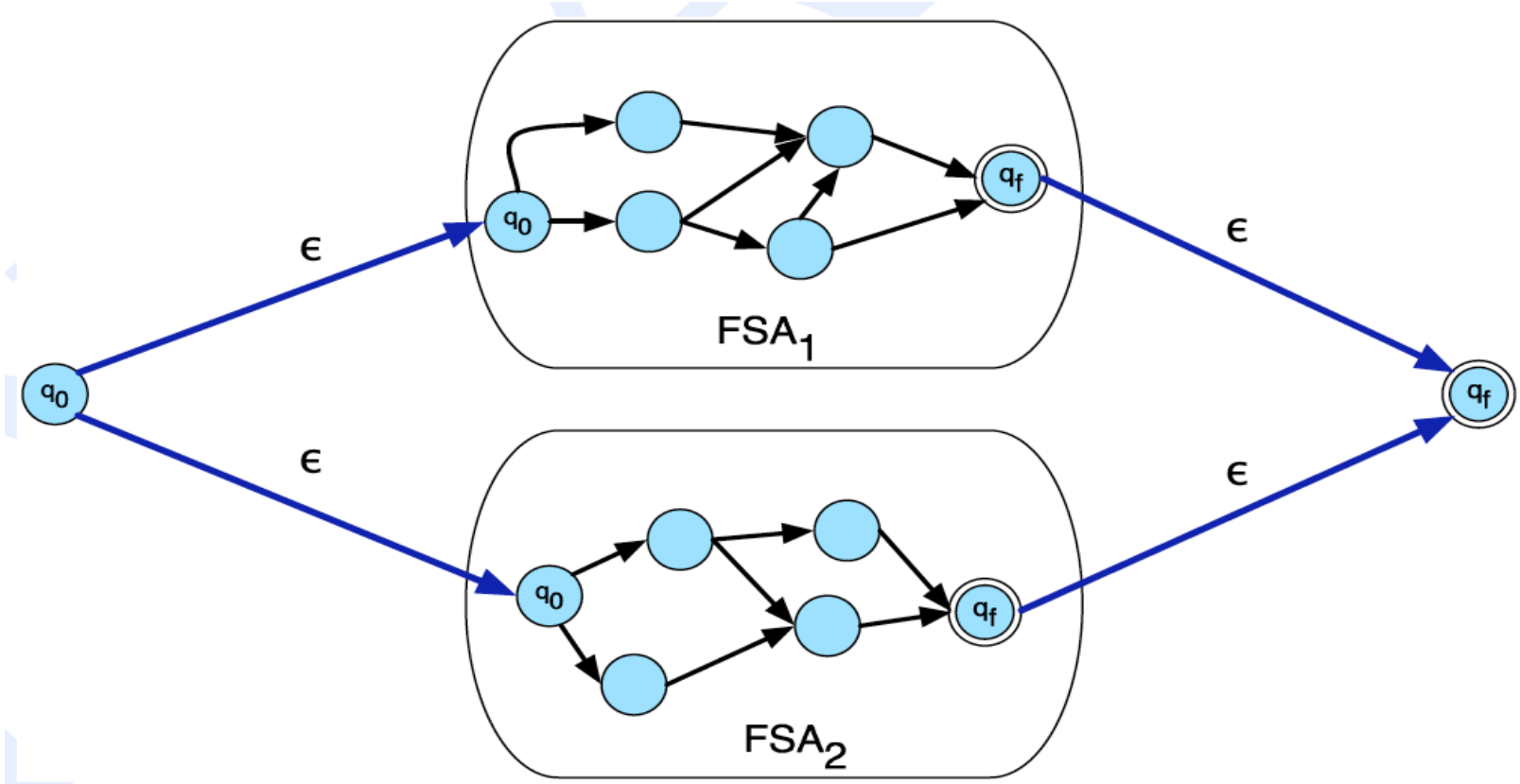
- I **could have danced** all night: MD VB VBN
- I **was dancing** when the lights went out: VBD VBG
- We **danced** the night away: VBD
- I **would have been dancing**, but …: MD VB VBN VBG
- He **has danced** his whole life: VBZ VBN
- She **dances** four times a week: VBZ
- He **loves to dance:** VBZ TO VB
- She **might dance** with him: MD VB



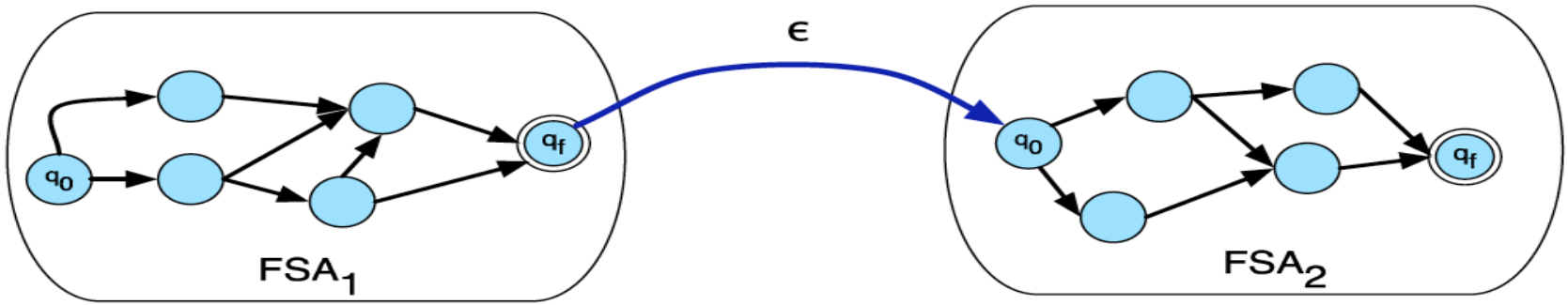
+ Compositional Machines

- Formal languages are just sets of strings
- Therefore, we can talk about various set operations (intersection, union, concatenation)
- This turns out to be a useful exercise

+ Union



Concatenation



+ Intersection

- Accept a string that is in both of two specified languages
- An indirect construction…
  - A^B = ~(~A or ~B)

(See details in SLP Ch 2)

Languages and Grammars

- We can model a language with a grammar
  - Production rules: LHS→ RHS
  - NonTerminals indicate a production rule can be applied
  - Terminals make up the “strings” (sentences) of the language
- The grammar defines all the possible strings of terminals in the language
  - A “language” is generally an infinite number of finite strings
  - Any string can be “accepted”/parsed by the grammar
  - The grammar can generate all the strings

Finite State Grammars for Speech

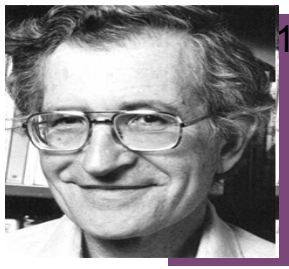

```
$takeoff_clearance : $cleared for takeoff [on] [$runway]
                   | takeoff [to the $direction]
$runway :          [runway] $runway_number;
$runway_number:    (zero | one | two) $digit;
$direction :       east | west | north | south;
```

- Elements
  - Rules:
  - Nonterminals: (Start with $)
  - Terminals: words
  - Expansions: the RHS
- Operators
  - Alternatives |
  - Optional [ ]
  - Repeat + *

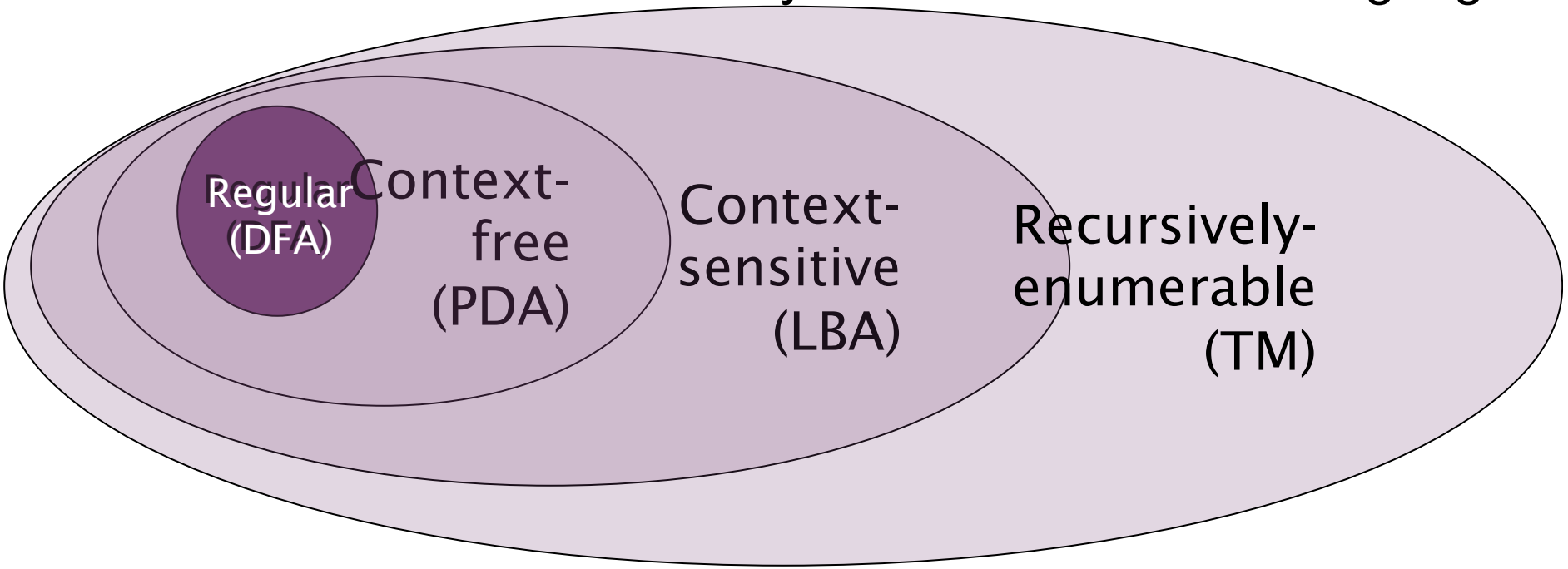
The “Generative Power” of Grammars

- Chomsky defined a hierarchy of language types distinguished by the characteristics of the grammars that can generate them
  - Finite State: A → Ab | b
  - Context Free: A → AB | a | b
  - Context Sensitive: bAc → bac
  - Recursively enumerable: No restrictions
- Many other important properties

The Chomsky Hierachy



- A containment hierarchy of classes of formal languages



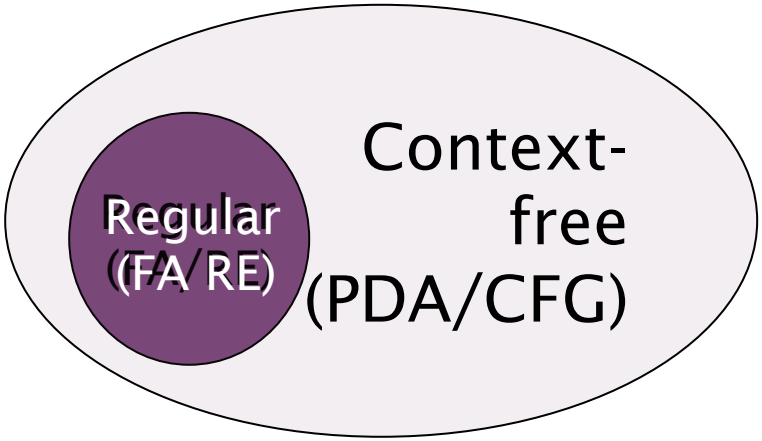
Chomsky Hierarchy

Level	Rules	Rule forms	Languages
Regular Finite state	A → Ab | b	LHS: 1 NT RHS: 1 NT, 1T or 1T	Eng verb group Pattern matching
Context Free	A → AB | a | b	LHS: 1 NT RHS: combo T & NT	"The dog the cat bit howled" XML
Context Sensitive	bAc → bac	LSH & RHS: NTs and Ts LHS < RHS	$A^nb^nc^n$ In Eng: "respectively"
Recursively enumerable	Any	Any	Any

Each level fully contains the level above it.

Context-Free Languages

- A language class larger than the class of regular languages
- Supports natural, recursive notation called "context-free grammar"
- Applications:
  - Parse trees, compilers
  - XML



An Example

- A palindrome is a word that reads identical from both ends
  - E.g., madam, redivider, malayalam, 010010010
- Let L = { w | w is a binary palindrome}
- Is L regular?
  - No.
  - Proof:
    - Let $w=0^N10^N$
    - By Pumping lemma, w can be rewritten as xyz, such that xy^kz is also L (for any $k\geq 0$)
    - But $|xy|\leq N$ and $y\neq\varepsilon$
    - ==> $y=0^+$
    - ==> xy^kz *will NOT* be in L for k=0
    - ==> Contradiction

But the language of palindromes…

is a CFL, because it supports recursive substitution (in the form of a CFG)

This is because we can construct a *grammar* like this:

Productions or rules

1. A ==> ε
2. A ==> 0
3. A ==> 1
4. A ==> 0A0
5. A ==> 1A1

Terminal

Variable or non-terminal

Same as:
A => 0A0 | 1A1 | 0 | 1 | ε

How does this grammar work?

How does the CFG for palindromes work?

An input string belongs to the language (i.e., accepted) iff it can be generated by the CFG

- Example: 01110
- G can generate this input string as follows:
  - A=> 0A0
    => 01A10
    => 01110

G:
A => 0A0 | 1A1 | 0 | 1 | ε

Context-Free Grammar: Definition

- A context-free grammar G=(V,T,P,S), where:
  - V: set of variables or non-terminals
  - T: set of terminals (this is equal to the alphabet)
  - P: set of *productions,* each of which is of the form V ==> α_1 | α_2 | …
    - Where each α_i is an arbitrary string of variables and terminals
  - S ==> start variable

CFG for the language of binary palindromes:

- G=({A},{0,1},P,A)
- P: A ==> 0 A 0 | 1 A 1 | 0 | 1 | ε

More examples

- Parenthesis matching in code
- Syntax checking
- In scenarios where there is a general need for:
  - Matching a symbol with another symbol, or
  - Matching a count of one symbol with that of another symbol, or
  - Recursively substituting one symbol with a string of other symbols

+ Tag-Markup Languages

- Roll ==> <ROLL> Class Students </ROLL>
- Class ==> <CLASS> Text </CLASS>
- Text ==> Char Text | Char
- Char ==> a | b | … | z | A | B | .. | Z
- Students ==> Student Students | ε
- Student ==> <STUD> Text </STUD>